

Problem 1.43

[Difficulty: 2]

1.43 A can of pet food has the following internal dimensions: 102 mm height and 73 mm diameter (each ± 1 mm at odds of 20 to 1). The label lists the mass of the contents as 397 g. Evaluate the magnitude and estimated uncertainty of the density of the pet food if the mass value is accurate to ± 1 g at the same odds.

Given: Pet food can

$$H = 102 \pm 1 \text{ mm (20 to 1)}$$

$$D = 73 \pm 1 \text{ mm (20 to 1)}$$

$$m = 397 \pm 1 \text{ g (20 to 1)}$$

Find: Magnitude and estimated uncertainty of pet food density.

Solution: Density is

$$\rho = \frac{m}{V} = \frac{m}{\pi R^2 H} = \frac{4}{\pi} \frac{m}{D^2 H} \quad \text{or} \quad \rho = \rho(m, D, H)$$

From uncertainty analysis:
$$u_\rho = \pm \left[\left(\frac{m}{\rho} \frac{\partial \rho}{\partial m} u_m \right)^2 + \left(\frac{D}{\rho} \frac{\partial \rho}{\partial D} u_D \right)^2 + \left(\frac{H}{\rho} \frac{\partial \rho}{\partial H} u_H \right)^2 \right]^{\frac{1}{2}}$$

Evaluating:
$$\begin{aligned} \frac{m}{\rho} \frac{\partial \rho}{\partial m} &= \frac{m}{\rho} \frac{4}{\pi} \frac{1}{D^2 H} = \frac{1}{\rho} \frac{4m}{\pi D^2 H} = 1; & u_m &= \frac{\pm 1}{397} = \pm 0.252\% \\ \frac{D}{\rho} \frac{\partial \rho}{\partial D} &= \frac{D}{\rho} (-2) \frac{4m}{\pi D^3 H} = (-2) \frac{1}{\rho} \frac{4m}{\pi D^2 H} = -2; & u_D &= \frac{\pm 1}{73} = \pm 1.37\% \\ \frac{H}{\rho} \frac{\partial \rho}{\partial H} &= \frac{H}{\rho} (-1) \frac{4m}{\pi D^2 H^2} = (-1) \frac{1}{\rho} \frac{4m}{\pi D^2 H} = -1; & u_H &= \frac{\pm 1}{102} = \pm 0.980\% \end{aligned}$$

Substituting:
$$\begin{aligned} u_\rho &= \pm \left[(1 \times 0.252)^2 + (-2 \times 1.37)^2 + (-1 \times 0.980)^2 \right]^{\frac{1}{2}} \\ u_\rho &= \pm 2.92\% \end{aligned}$$

$$V = \frac{\pi}{4} D^2 H = \frac{\pi}{4} \times (73)^2 \text{ mm}^2 \times 102 \text{ mm} \times \frac{\text{m}^3}{10^9 \text{ mm}^3} = 4.27 \times 10^{-4} \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{397 \text{ g}}{4.27 \times 10^{-4} \text{ m}^3} \times \frac{\text{kg}}{1000 \text{ g}} = 930 \text{ kg/m}^3$$

Thus:
$$\rho = 930 \pm 27.2 \text{ kg/m}^3 \text{ (20 to 1)}$$